2017 KIAS-SNU Physics Winter Camp

# Gravitational Waves and Compact Stars

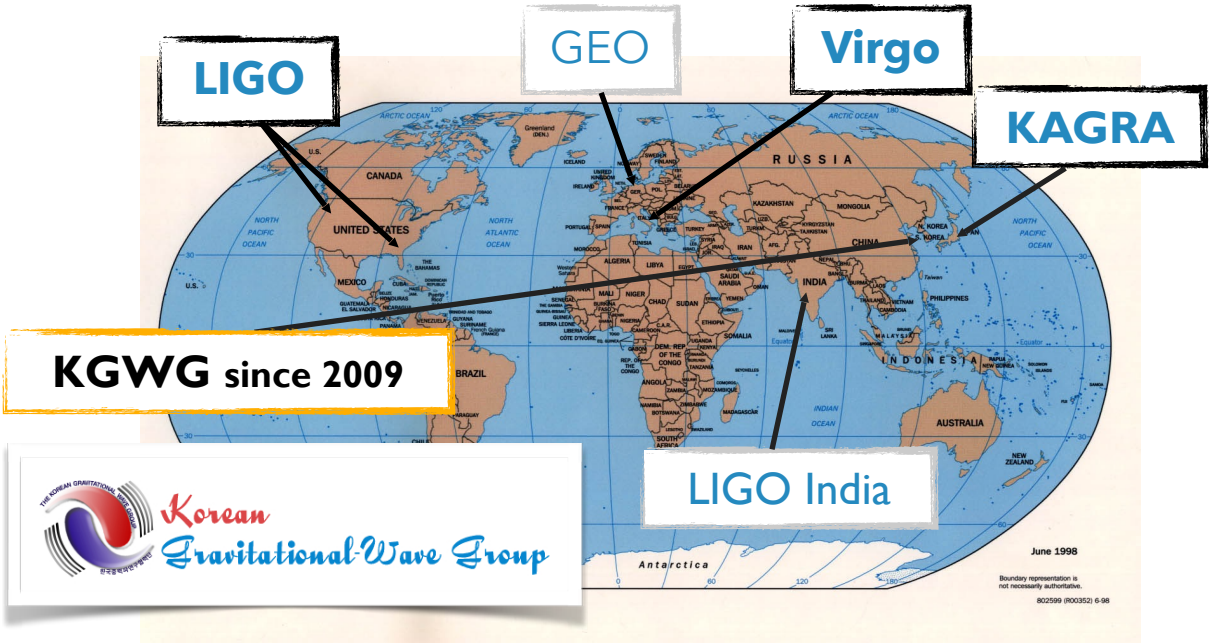
Chang-Hwan Lee / Pusan National University



## Contents


- **Motivation**
- Spacetime and GW
- Principle of general relativity & GW
- LIGO
- Neutron star coalescence and GW
- Prospects

# Network of GW Interferometers

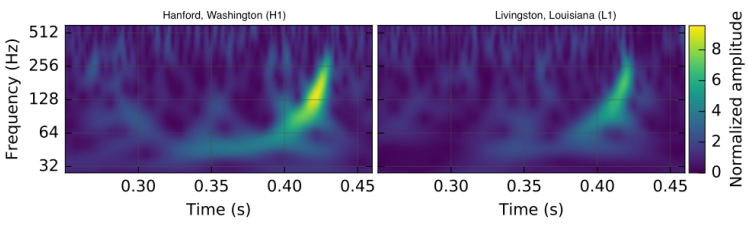


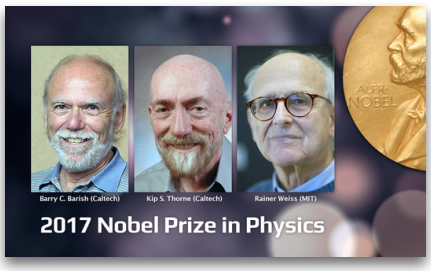



# LIGO

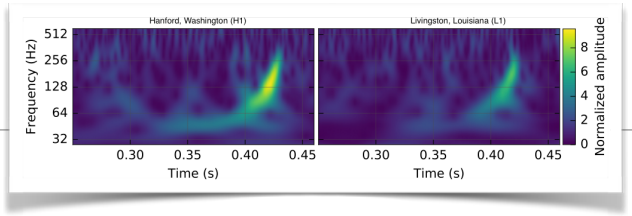
- 2015 First detection of gravitational waves
  - First detection of black hole binary
  - First detection of heavy (stellar-mass) black holes
  - Opens a new era of GW Astronomy

- 2017 Nobel Prize in Physics



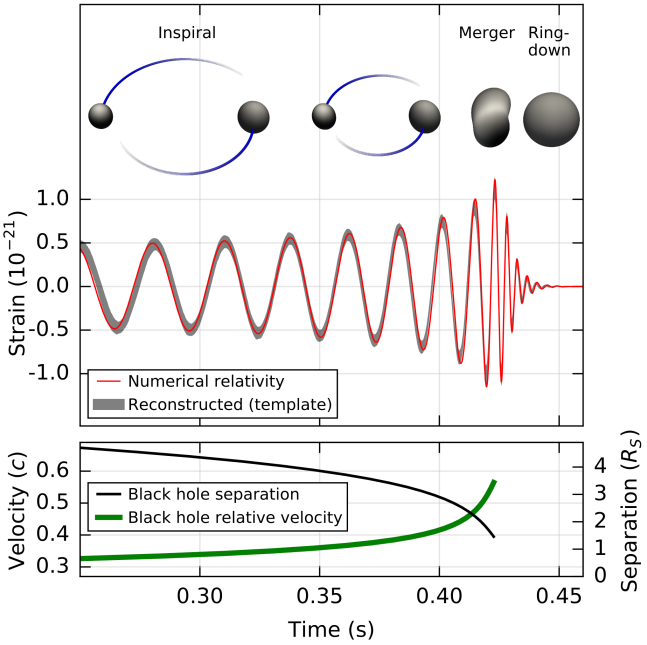
# Gw150914







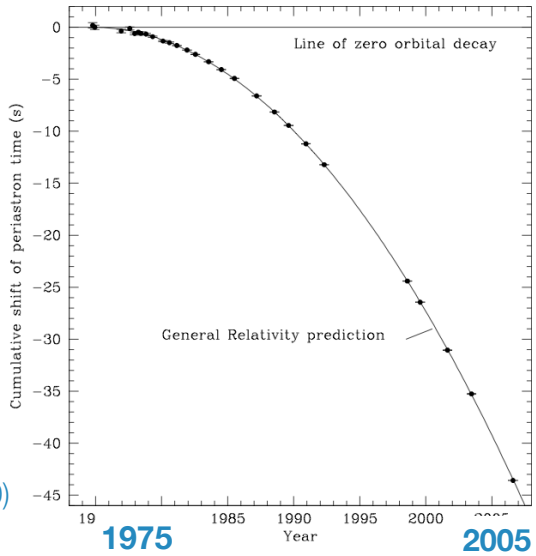
LSC LIGO Scientific Collaboration VIRGO






## Gravitational waves from neutron star binaries

- B1913+16 / Hulse & Taylor (1975)
- Orbital period change due to GW radiation
- 1993 Nobel Prize
- LIGO is based on NS binary mergers
- **Why 2019 ?**
  ***d = O(100 Mpc)***



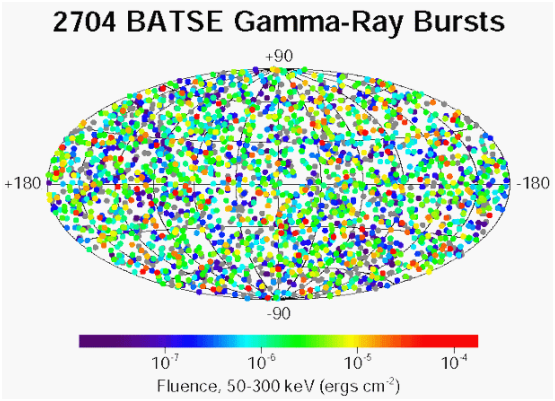

Weisberg, Nice, Taylor, ApJ (2010)

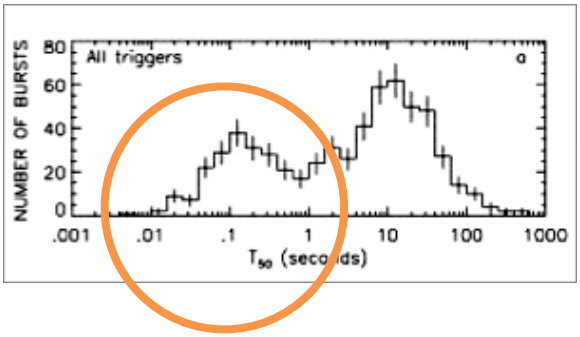
# **NS (radio pulsar)** which will coalesce within Hubble time

| PSR | $P$ (ms) | $P_b$ (hr) | $e$ | Total Mass $M_\odot$ | $\tau_c$ (Myr) | $\tau_{GW}$ (Myr) | |
|---|---|---|---|---|---|---|---|
| J0737−3039A | 22.70 | 2.45 | 0.088 | 2.58 | 210 | 87 | (2003) |
| J0737−3039B | 2773 | 2.45 | 0.088 | 2.58 | 50 | 87 | (2004) |
| B1534+12 | 37.90 | 10.10 | 0.274 | 2.75 | 248 | 2690 | (1990) |
| J1756−2251 | 28.46 | 7.67 | 0.181 | 2.57 | 444 | 1690 | (2004) |
| B1913+16 | 59.03 | 7.75 | 0.617 | 2.83 | 108 | 310 | (1975) |
| B2127+11C | 30.53 | 8.04 | 0.681 | 2.71 | 969 | 220 | (1990) |
| J1141−6545† | 393.90 | 4.74 | 0.172 | 2.30 | 1.4 | 590 | (2000) |



# **sGRB** short-hard gamma-ray bursts from NS mergers

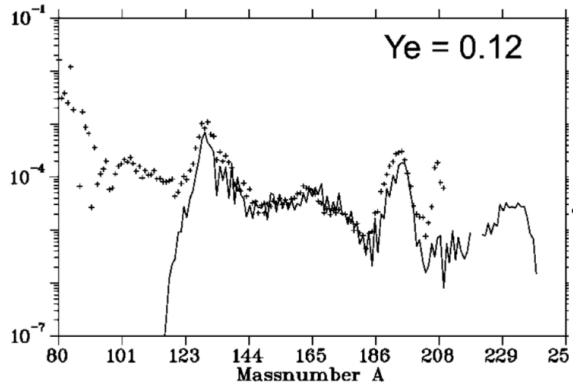


# Neutron Star Mergers / Kilonova

- **Gravitational Waves**
- **Sources of Heavy Elements**

S Rosswog 2015



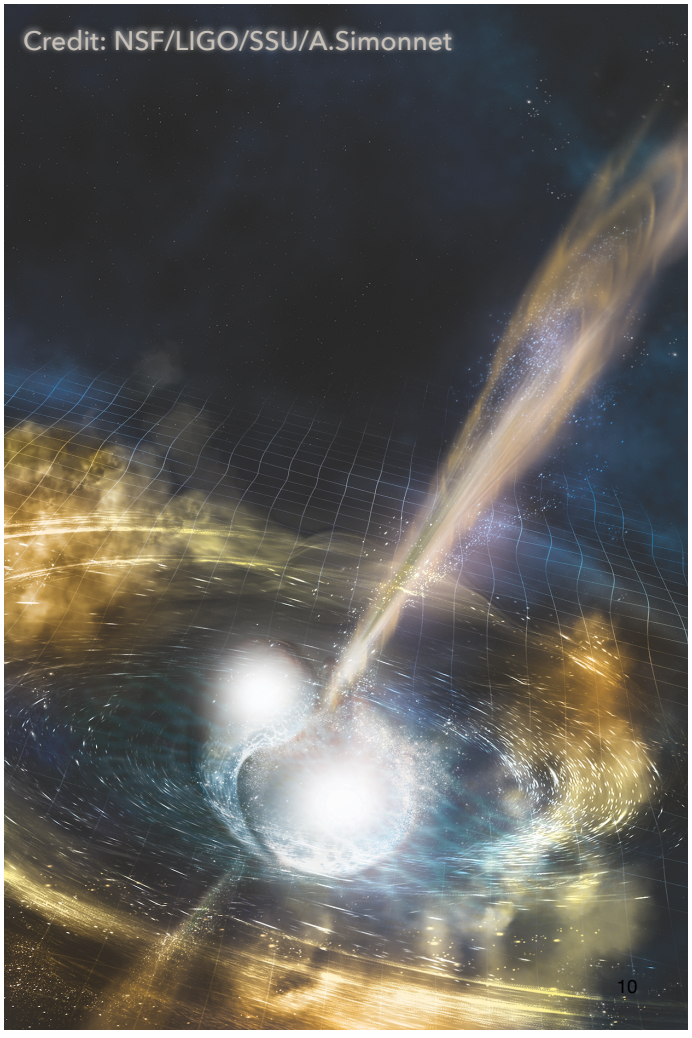

solar pattern vs NS-merger

- **Supernovae:**
  neutrino-driven wind
  r-process **peak at A~130**
- **NS mergers:**
  r-process **peak at A~195**

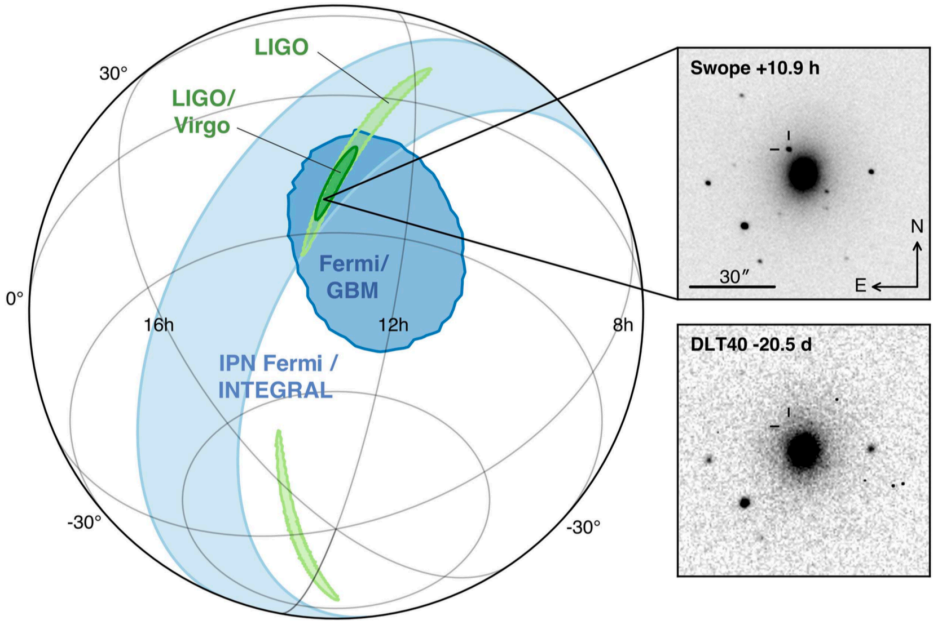


Press Release Oct 16, 2017
GW from Binary NS Mergers

*GW 170817 (**d=40 Mpc**)*
*GRB 170817A by Fermi-GBM*
*Kilonova/X-ray/Optical Afterglows*



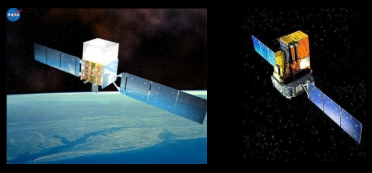
# GW170817 / GRB170817A



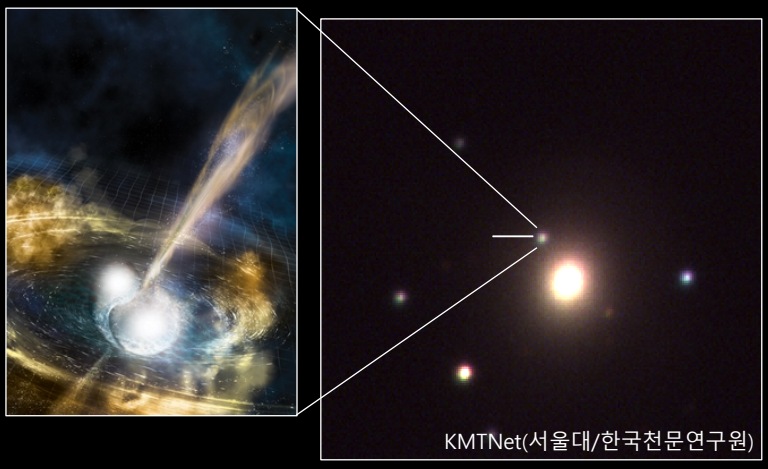


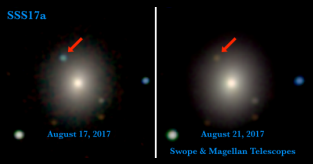



2017-08-17 12:41:04 UTC
라이고 및 비르고 중력파 신호 포착

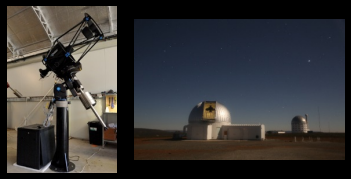


+2초 후
페르미 및 인티그랄 감마선 신호 포착




+약 11시간 후
칠레 천문대 망원경들이
가시광선 신호 포착



+약 21시간 후
국내연구진 호주 이상각망원경으로
추적관측시작. 이후 약 4주간 추적관측
(KMTNet, BOOTES-5망원경 등)




+약 9일 후
찬드라 우주망원경
X-선 신호 포착

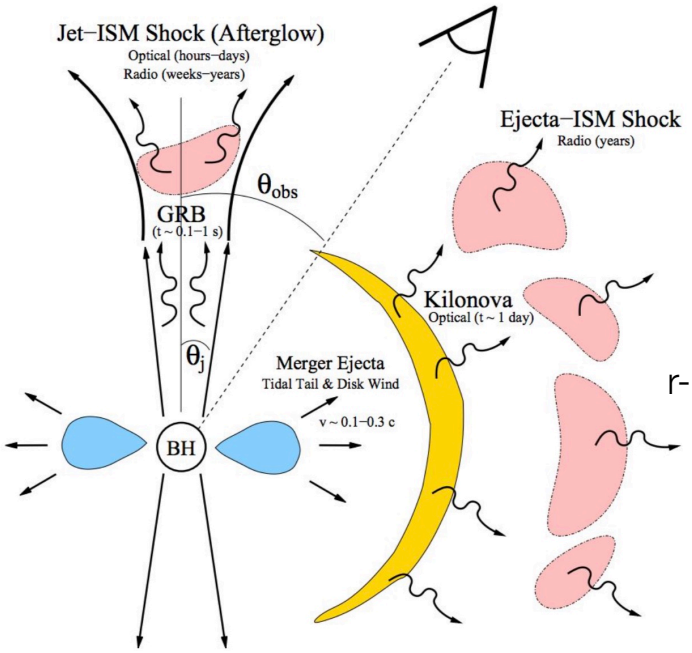


+약 16일 후
지상 전파망원경
전파 신호 포착

그림출처: LIGO, NASA, 1M2H/UC Santa Cruz and Carnegie Observatories/Ryan Foley,
서울대/한국천문연구원/임명신, NSF/Sonoma 주립대/A. Simonnet, NRAO

서울대학교 초기우주천체연구단
(임명신)

# GRB and Kilonova




r-process **peak at A~195**

Metzger and Berger 2012



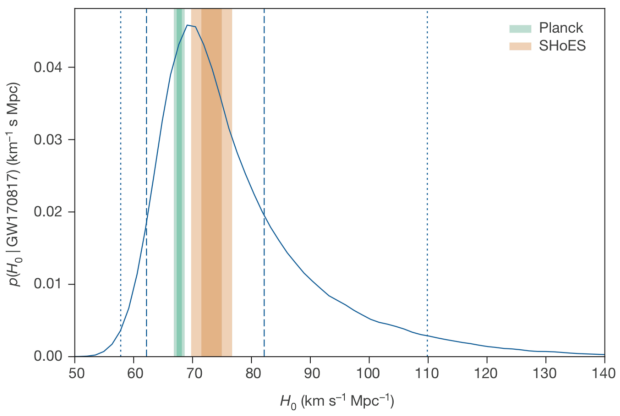
LETTER

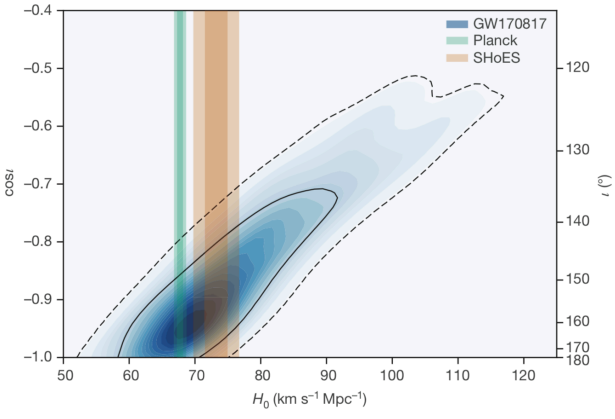
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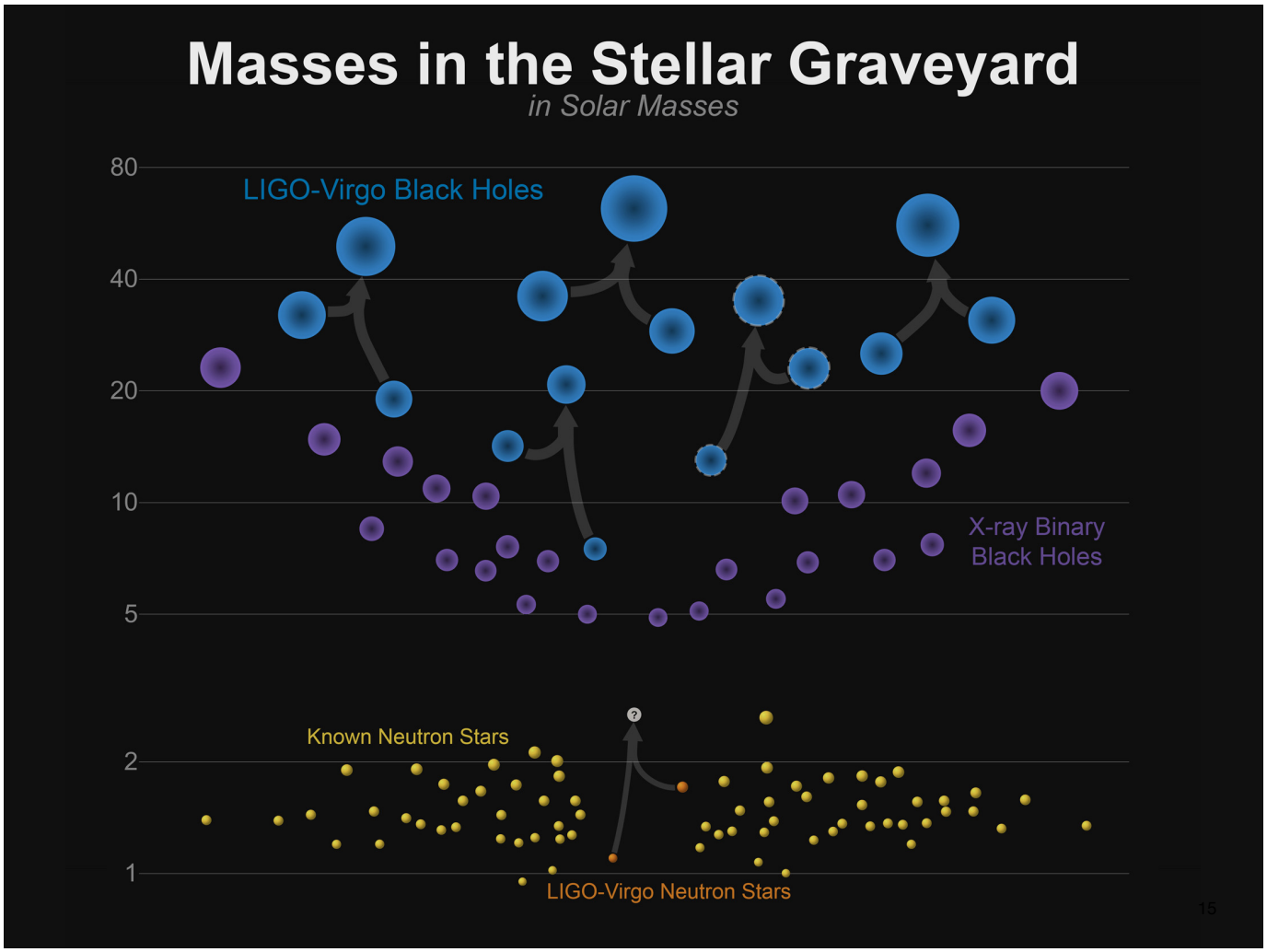
## A gravitational-wave standard siren measurement of the Hubble constant

The LIGO Scientific Collaboration and The Virgo Collaboration*, The 1M2H Collaboration*, The Dark Energy Camera GW-EM Collaboration and the DES Collaboration*, The DLT40 Collaboration*, The Las Cumbres Observatory Collaboration*, The VINROUGE Collaboration* & The MASTER Collaboration*

$$H_0 = 70.0^{+12.0}_{-8.0}\,\mathrm{km\ s^{-1}Mpc^{-1}}$$

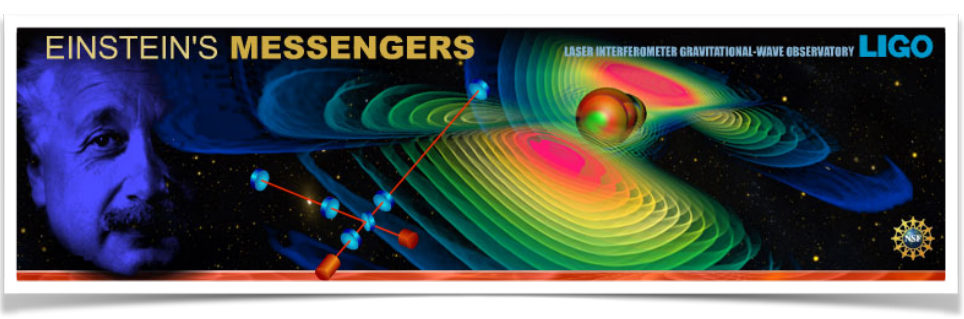





## Gravitational-Wave & Multi-Messenger Astronomy

- First direct detection of GW in 2015
- First detection of BHs with masses 30 ~ 60 solar mass
- GW, Gamma-ray, Optical, X-ray, Radio detection from NS mergers
- New era for GW Astronomy & Multi-Messenger Astronomy

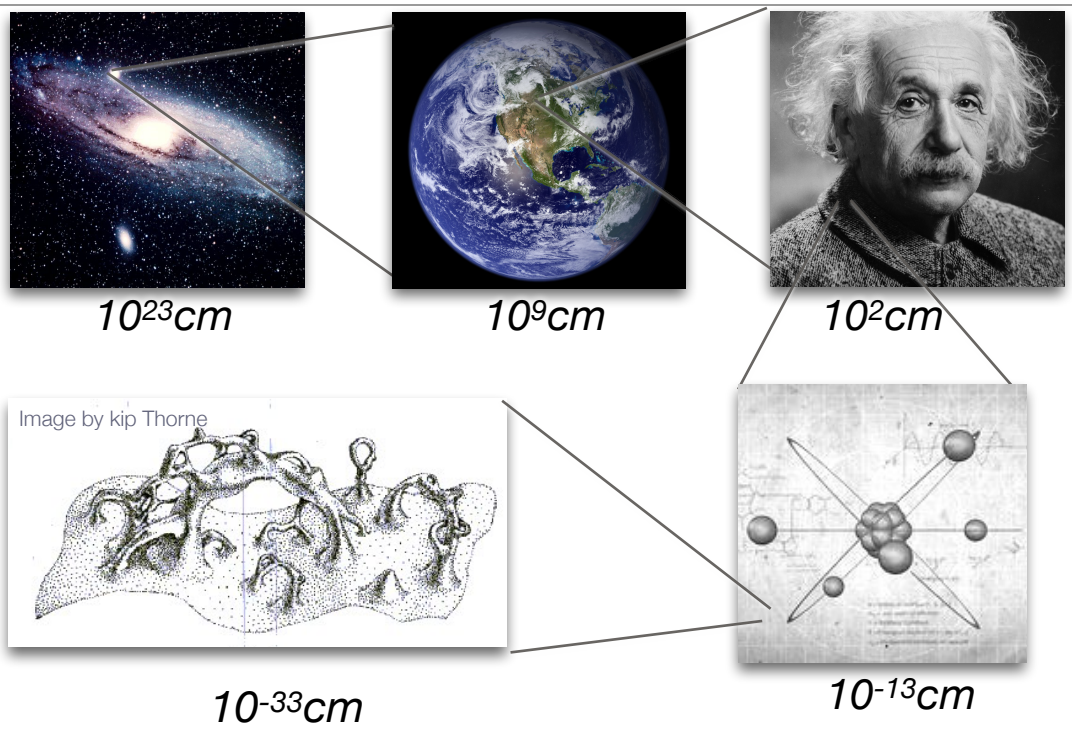


# Contents

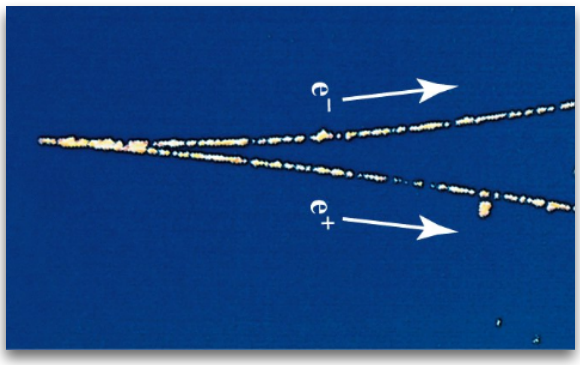
- Motivation
- **Spacetime and GW**
- Principle of general relativity & GW
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- Neutron star coalescence and GW
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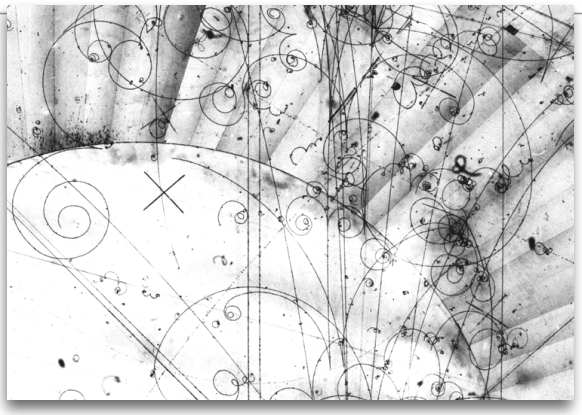
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# 진공은 무엇인가

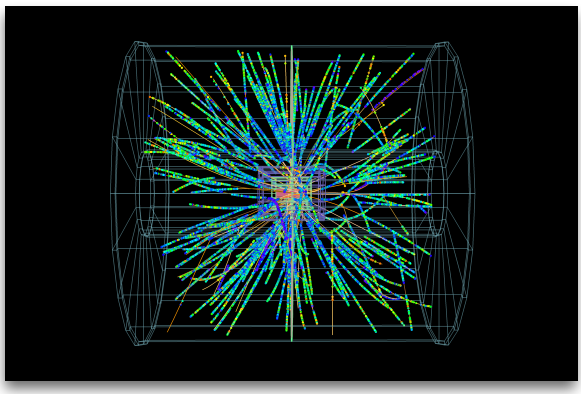


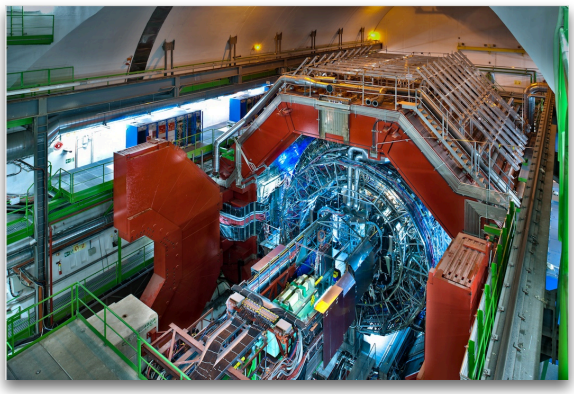

- 빛은 사라지고
  입자-반입자 쌍 생성
- 입자-반입자는 어디에서
- `없다`는 과연 무엇인가





# 진공은 무한한 창조의 공간





LHC@CERN

## 우리는 왜 진공을 인식하지 못하는가

- 우리는 큰 변화 만을 인식한다
- 진공 자체도 변할 수 있다

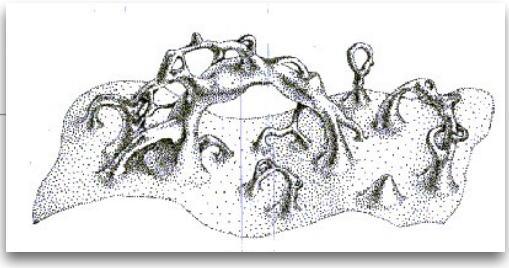
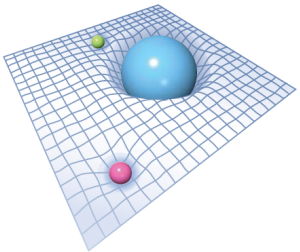


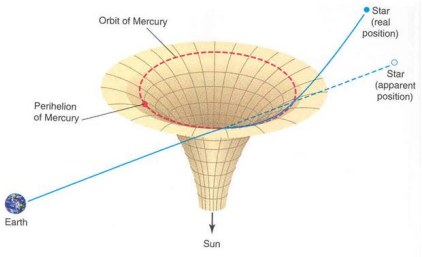
Image by kip Thorne



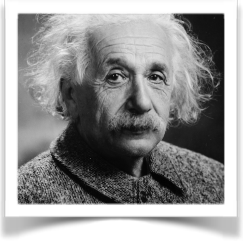
## 시공간의 변화 / 일반상대성이론 1915

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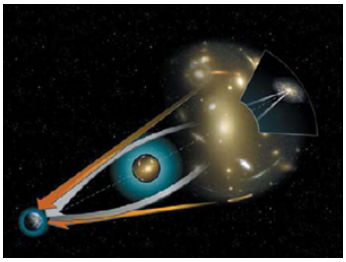


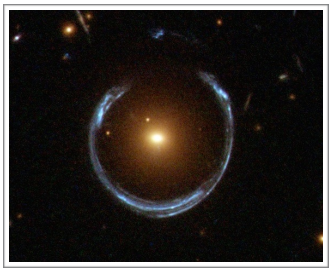



**중력이 있으면 빛도 휜다**

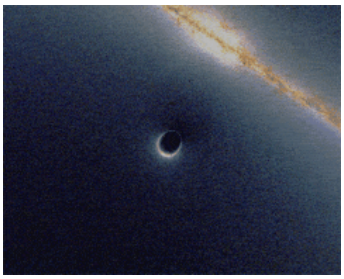


# 중력렌즈





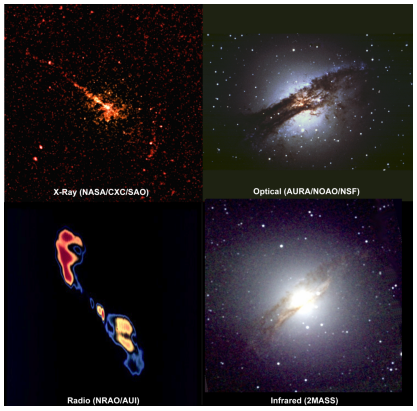
NASA/ESA



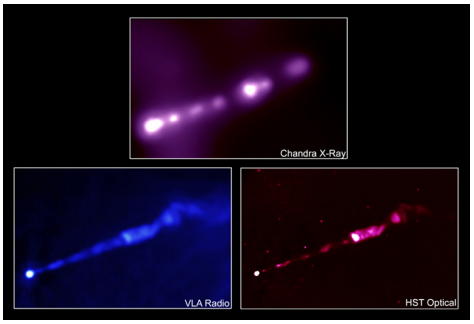
마이크로렌징 / 외계행성탐색



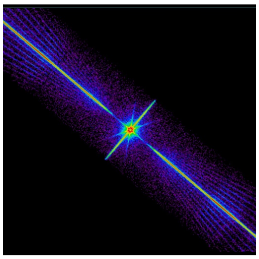
# 블랙홀 발견




Centaurus A

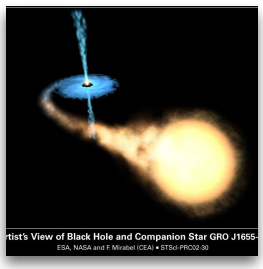


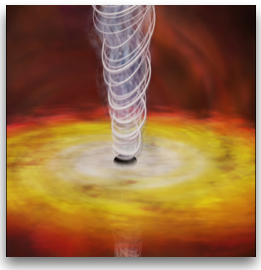

M87



XTEJ1118+480

- 은하중심 거대 블랙홀
- 블랙홀 쌍성 / 강한 엑스선방출
- 2002 노벨물리학상



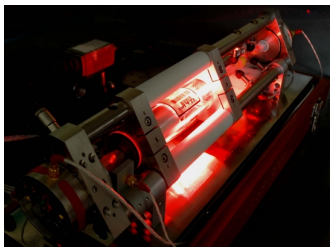





## 빛으로 이해하는 우주 / 시공간 자체의 변화

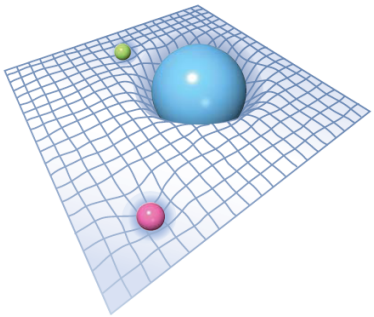
시간(세슘-133원자)과 길이의 기준
시간과 3차원 공간, 그리고 물질은 광속불변의 법칙에 의해 얽혀 있음

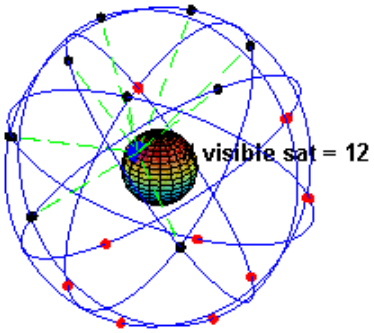






## 중력과 시간지연





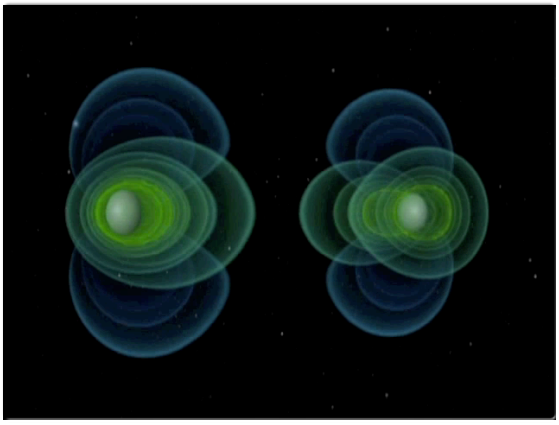

GPS

에너지(질량)는 시간의 흐름을 변화시킨다

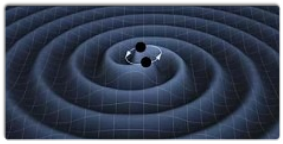
중력은 시공간 자체의 변화

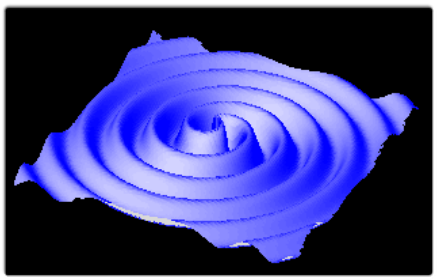
# 중력파

블랙홀의 충돌



시공간의 잔물결
빛의 속도로 이동

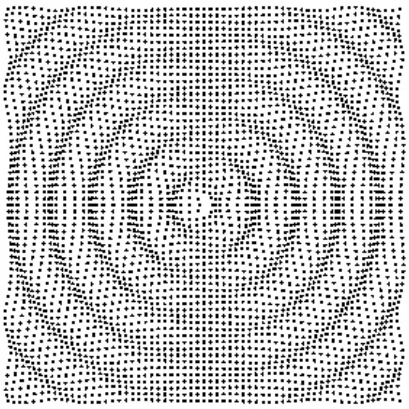




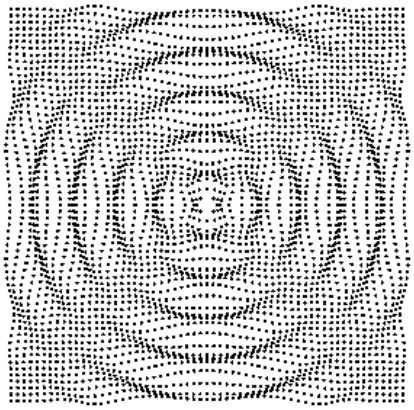
LSC LIGO Scientific Collaboration
VIRGO



# 전자기파/중력파

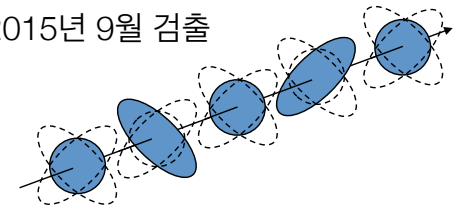


전자기파
맥스웰 1867년 예측
헤르츠 1886-91 검출



중력파
아인슈타인 1916년 예측
라이고 2015년 9월 검출





# Contents

- Motivation
- Spacetime and GW
- **Principle of general relativity & GW**
- LIGO
- Neutron star coalescence and GW
- Prospects



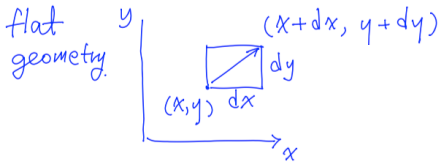
# References


- **J.M. Hartley, Gravity An Introduction to Einstein's General Relativity**
- S.M. Carroll, Spacetime and Geometry

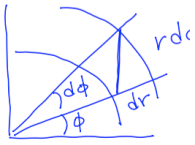
- **Lecture notes by K. Thorne**
  - https://www.lorentz.leidenuniv.nl/lorentzchair/thorne/Thorne1.pdf
  - http://www.pmaweb.caltech.edu/Courses/ph136/yr2012/ (together with R. Blandford, Chapters 25 & 27)
- Recent LIGO papers
  - Abbott et al. 2016, "Abbott et al. 2016, "Observation of Gravitational Waves from a Binary Black Hole Merger", 2016, PRL, 116, 061102
  - Abbott et al., 2016, "ASTROPHYSICAL IMPLICATIONS OF THE BINARY BLACK HOLE MERGER GW150914" , ApJL, 828, L22
  - Abbott et al. 2016, "GW150914: Implications for the Stochastic Gravitational-Wave Background from Binary Black Holes", PRL, 115, 131102
  - Abbott et al. 2016, "GW150914: The Advanced LIGO Detectors in the Era of First Discoveries", 2016, PRL, 116, 131103

# Line element (ds) : how to specify geometry?

- should be independent of coordinate transformation
- should be testable by a local measurement



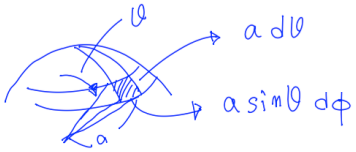

$$ds^2 = (dx)^2 + (dy)^2$$




$$ds^2 = (dr)^2 + (rd\phi)^2$$

$$dr, d\phi \ll 1$$

Spherical geometry

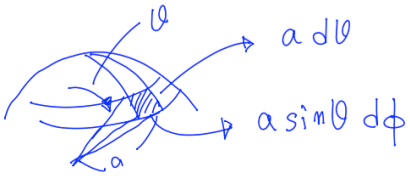



$$ds^2 = a^2(d\theta^2 + \sin^2\theta \cdot d\phi^2)$$

$$d\theta, d\phi \ll 1$$



more general in curved space $ds^2 = a^2(d\theta^2 + \sin^2\theta \cdot d\phi^2)$




$$ds^2 = a^2(d\theta^2 + f^2(\theta)d\phi^2)$$

$$C(\theta = \theta_0) = \int_0^{2\pi} af(\theta_0)d\phi = 2\pi a f(\theta_0)$$

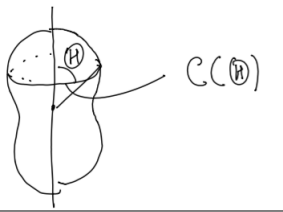
$$r = a\int_0^{\theta_0} d\theta = a\theta_0$$

$$C = 2\pi a f(r/a)$$

e.g) Surface of Peanut shape

$f(\theta) = \sin\theta\left(1 - \frac{3}{4}\sin^2\theta\right)$

$C = 2\pi a \sin(H)\left(1 - \frac{3}{4}\sin^2(H)\right)$



# line element in special/general relativity

**special relativity (line element is invariant under Lorentz transformation)**

$$\begin{aligned} ds^2 &= -(cdt)^2 + dx^2 + dy^2 + dz^2 \\ &= -(cdt)^2 + dr^2 + r^2(d\theta^2 + \sin^2\theta \; d\phi^2) \end{aligned}$$

**general relativity (educated guess)**

$$ds^2 = g_{\alpha\beta} dx^\alpha dx^\beta \qquad (\alpha, \beta = 0, 1, 2, 3)$$

**information of gravity (metric)**

**flat space time**

$$ds^2 = \eta_{\alpha\beta} dx^\alpha dx^\beta$$

$$\eta_{\alpha\beta} = \mathrm{diag}(-1, 1, 1, 1) = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$



# notations

$$a \cdot b = a_\alpha b^\alpha = g_{\beta\alpha} a^\beta b^\alpha = g_{\alpha\beta} a^\beta b^\alpha$$

$$a_\alpha = g_{\alpha\beta} a^\beta$$

$$g^{\alpha\gamma} g_{\gamma\beta} = \delta^\alpha_\beta$$

$$a^\alpha = g^{\alpha\beta} a_\beta$$

# **proper time** under curved geometry

$$ds^2 = g_{\alpha\beta} dx^\alpha dx^\beta \qquad d\tau^2 = -ds^2$$

$$\frac{ds^2}{d\tau^2} = -1 = g_{\alpha\beta} \frac{dx^\alpha}{d\tau} \frac{dx^\beta}{d\tau} = g_{\alpha\beta} u^\alpha u^\beta$$

**quantum fields “move” along the trajectory which minimises the proper time under curved geometry**

**Special Relativity**

$$-\frac{ds^2}{d\tau^2} = \left(\frac{dt}{d\tau}\right)^2 - \left(\frac{d\vec{x}}{cd\tau}\right)^2 = 1$$

$$(mc^2)^2 \left(\frac{dt}{d\tau}\right)^2 - (mc)^2 \left(\frac{dt}{d\tau}\frac{d\vec{x}}{dt}\right)^2 = (mc^2)^2$$

$$\gamma = \frac{dt}{d\tau} \qquad E = \gamma mc^2 \qquad p = \gamma mv$$

$$E^2 - (pc)^2 = (mc^2)^2$$



# Geodesic equation / extremize proper time

**Flat space**

$$\delta \int \sqrt{-\eta_{\alpha\beta} dx^\alpha dx^\beta} = 0 \qquad \frac{d^2 x^\alpha}{d\tau^2} = 0$$

**Newtonian**

$$\delta \int \sqrt{\left(1 + \frac{2\Phi}{c^2}\right)(cdt)^2 - \left(1 - \frac{2\Phi}{c^2}\right)(d\vec{x})^2} = 0 \qquad \frac{d^2 x^i}{dt^2} = -\frac{\partial \Phi}{\partial x^i}$$

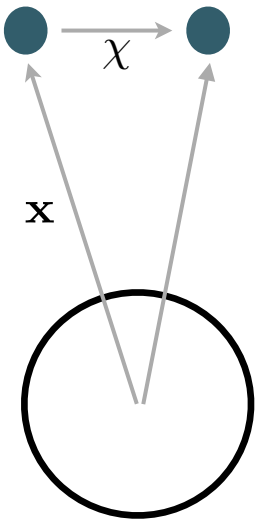
**General**

$$\delta \int \sqrt{-g_{\alpha\beta} dx^\alpha dx^\beta} = 0 \qquad \frac{d^2 x^\alpha}{d\tau^2} = -\Gamma^\alpha_{\beta\gamma} \frac{dx^\beta}{d\tau} \frac{dx^\gamma}{d\tau}$$

**Christoffel symbol**

# Tidal gravitational force / Newtonian case

$$\frac{d^2x^i}{dt^2} = -\delta^{ij}\frac{\partial\Phi(\mathbf{x})}{\partial x^j}\,, \qquad \frac{d^2(x^i+\chi^i)}{dt^2} = -\delta^{ij}\frac{\partial\Phi(\mathbf{x}+\chi)}{\partial x^j}$$




$$\frac{\partial\Phi(\mathbf{x}+\chi)}{\partial x^j} \approx \frac{\partial\Phi(\mathbf{x})}{\partial x^j} + \frac{\partial}{\partial x^k}\left(\frac{\partial\Phi(\mathbf{x})}{\partial x^j}\right)\chi^k + \ldots$$

$$\frac{d^2\chi^i}{dt^2} = -\delta^{ij}\left(\frac{\partial^2\Phi}{\partial x^j\partial x^k}\right)_{\mathbf{x}}\chi^k$$

**tidal acceleration tensor**



# Einstein's Field Equation

- In Newtonian dynamics: Poisson's equation:

$$\nabla^2\Phi = \delta^{ij}\left(\frac{\partial^2\Phi}{\partial x^i\partial x^j}\right) = 4\pi G\rho$$

  i.e., summation over the quantities that describe the geodesic deviation (=tidal acceleration tensor).

- The Ricci tensor is similarly defined with **Φ**, i.e., summation of all tidal components.

- In the presence of energy (mass, etc.), Einstein's field equation becomes

$$R_{\alpha\beta} - \frac{1}{2}g_{\alpha\beta}R = \frac{8\pi G}{c^4}T_{\alpha\beta}$$

Energy-momentum tensor

Ricci tensor $R_{\mu\nu} = R^{\lambda}_{\mu\lambda\nu}$

Curvature scalar $R = R^{\mu}_{\mu}$

# Einstein's field equation with source

$$G_{\alpha\beta} = \frac{8\pi G}{c^4} T_{\alpha\beta} \qquad G_{\alpha\beta} \equiv R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R$$

$$T^{\alpha\beta} = \begin{pmatrix} \text{energy density} & \text{energy flux} \\ \text{mom. endsity} & \text{stress tensor} \end{pmatrix}$$

$$\nabla_\beta T^{\alpha\beta} = 0 \longrightarrow \nabla_\beta G^{\alpha\beta} = 0$$

**local conservation of energy-momentum**

**Bianchi identity**

**Newtonian limit**



# Newtonian vs Einstein gravity

$$\frac{ds^2}{d\tau^2} = -1 = g_{\alpha\beta} \frac{dx^\alpha}{d\tau} \frac{dx^\beta}{d\tau} = g_{\alpha\beta} u^\alpha u^\beta$$

**What mass does**

$$\vec{F} = -m\vec{\nabla}\Phi$$

produce a field

$$ds^2 = g_{\alpha\beta}(x) dx^\alpha dx^\beta$$

curves spacetime

**Motion of a particle**

$$\frac{d^2 x^i}{dt^2} = -\delta^{ij} \frac{\partial \Phi}{\partial x^j}$$

Newton's law of motion

$$\frac{d^2 x^\alpha}{d\tau^2} = -\Gamma^\alpha_{\beta\gamma} \frac{dx^\beta}{d\tau} \frac{dx^\gamma}{d\tau}$$

Geodesic equation

**Field equation**

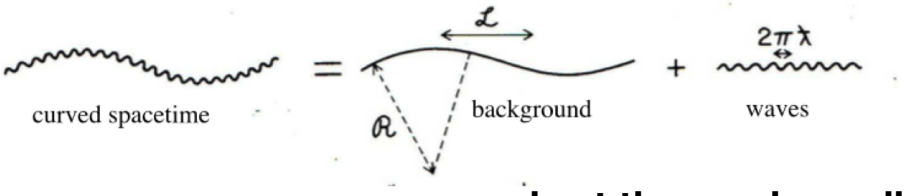
$$\nabla^2 \Phi = 4\pi G \mu$$

$$G_{\alpha\beta} = \frac{8\pi G}{c^4} T_{\alpha\beta}$$

# Principles of Gravitational Waves

$c = 1$ unit

**long timescale change**




**short timescale oscillation**



# Linearized Vacuum Einstein Equation

$$g_{\alpha\beta} = \eta_{\alpha\beta} + h_{\alpha\beta}$$

$$|h_{\mu\nu}| \ll 1$$

$$\eta_{\alpha\beta} = \mathrm{diag}(-1,1,1,1) = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\delta R_{\alpha\beta} = \frac{1}{2}\left[-\Box h_{\alpha\beta} + \partial_\alpha V_\beta + \partial_\beta V_\alpha\right] = 0$$

$$\Box \equiv \eta^{\alpha\beta}\partial_\alpha\partial_\beta = -\frac{\partial^2}{\partial t^2} + \vec{\nabla}^2$$

$$V_\alpha \equiv \partial_\gamma h^\gamma_\alpha - \frac{1}{2}\partial_\alpha h^\gamma_\gamma$$

$$h^\gamma_\alpha = \eta^{\gamma\delta} h_{\delta\alpha}$$

**Gauge transformation**

$$x'^\alpha = x^\alpha + \xi^\alpha(x)$$

$$h'_{\alpha\beta} = h_{\alpha\beta} - \partial_\alpha\xi_\beta - \partial_\beta\xi_\alpha$$

**Lorentz gauge**

$$V'_\alpha \equiv \partial_\beta h'^\beta_\alpha - \frac{1}{2}\partial_\alpha h'^\beta_\beta = 0$$

**Field equation**

$$\Box h'_{\alpha\beta} = 0$$

# EM waves & Gravitational waves

| **Maxwell equation** | **Einstein equation** |
|---|---|
| source (charge, spin) | source (energy, …) |
| E,M fields | space-time curvature (gravity) |
| EM waves | gravitational waves |



# Linearized Gravitation vs Electromagnetism

| | |
|---|---|
| **basic potential** $h_{\alpha\beta}(x)$ | $\left(\Phi(t,\vec{x}), \vec{A}(t,\vec{x})\right)$ |
| **fields** $\delta R_{\alpha\beta\gamma\delta}(x)$ | $\vec{E}(t,\vec{x}), \vec{B}(t,\vec{x})$ |
| **Gauge transformation** $h_{\alpha\beta} \to h_{\alpha\beta} - \partial_\alpha \xi_\beta - \partial_\beta \xi_\alpha$ | $\vec{A} \to \vec{A} + \vec{\nabla}\Lambda$<br>$\Phi \to \Phi - \partial\Lambda/\partial t$ |
| **Lorentz gauge** $\partial_\beta h^\beta_\alpha - \frac{1}{2}\partial_\alpha h^\beta_\beta = 0$ | $\vec{\nabla}\cdot\vec{A} + \partial\Phi/\partial t = 0$ |
| **field equation** $\Box h_{\alpha\beta} = 0$ | $\Box\vec{A} = 0$<br>$\Box\Phi = 0$ |

# Transverse-traceless (TT) gauge

**impose Lorentz gauge**

$$\Box h_{\alpha\beta} = 0$$

**wave equation**

**further impose TT gauge**

$$h_{\alpha\beta} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \mathrm{a} & \mathrm{b} & 0 \\ 0 & \mathrm{b} & \text{-a} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} e^{i(\vec{k}\cdot\vec{x}-\omega t)}$$

$$a = \frac{1}{2}\left(h_{xx} - h_{yy}\right)$$

$$b = h_{xy}$$



# production of gravitational waves

**near source, one cannot assume tranverse-traceless form**

- keep entire perturbation / solve for gravitational wave
- impose TT gauge only far from the source

**trace-reversed perturbation**

$$\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2} h \eta_{\mu\nu}$$

**Lorentz gauge**

$$\frac{\partial \bar{h}_{\mu\nu}}{\partial x_\nu} = 0$$

**wave-equation with sources**

$$\Box \bar{h}_{\mu\nu} = -16\pi G T_{\mu\nu}$$

# Further properties

**in vacuum**

$$\Box \bar{h}_{\mu\nu} = 0$$

**plane waves propagating with speed of light**

$$\bar{h}_{\mu\nu} = Re\{A_{\mu\nu} \exp(ik_\lambda x^\lambda)\}$$

**coordinate transformation**

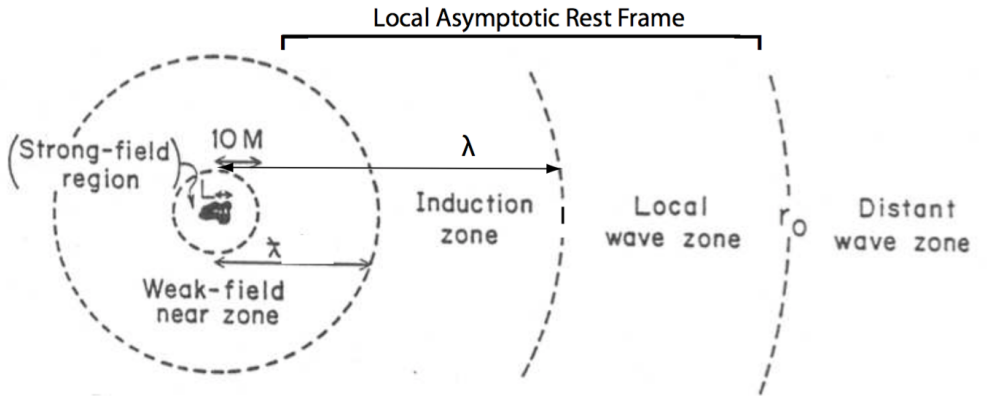
$$A^\mu_\mu = 0 \text{ (traceless)}, \quad A_{0_i} = 0 \text{ purely spatial}$$

**TT gauge far away from any sources**

$$\bar{h}^{TT}_{\mu\nu} = h^{TT}_{\mu\nu}$$



# Regions of space around a source of GW



# **GW** propagating in z-direction **in TT gauge**

**plus polarisation** **cross polarisation**

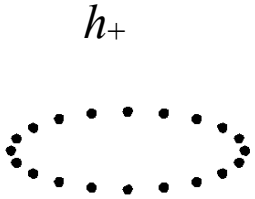
$$h^{TT}_{\mu\nu} = h_+(t-z)\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} + h_\times(t-z)\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

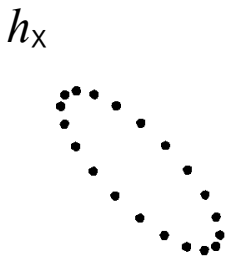
**line element**

$$ds^2 = -dt^2 + (1+h_+)dx^2 + (1-h_+)dy^2 + dz^2 + 2h_\times dxdy$$

**lengths in x- & y-directions**

$$L_x = \int_{x_1}^{x_2} \sqrt{1+h_+}dx \approx (1+\frac{1}{2}h_+)L_{x0};$$
$$L_y = \int_{y_1}^{y_2} \sqrt{1-h_+}dy \approx (1-\frac{1}{2}h_+)L_{y0}$$






# Generation of gravitational waves

$$\Box\bar{h}_{\mu\nu} = -16\pi G T_{\mu\nu}$$

$$\bar{h}_{\mu\nu}(t,\mathbf{x}) = 4\int \frac{T_{\mu\nu}(t-|\mathbf{x}-\mathbf{x}'|,\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|}d^3x \approx \frac{4}{r}\int T_{\mu\nu}(t-|\mathbf{x}-\mathbf{x}'|,\mathbf{x}')d^3x$$

$|\mathbf{x}'| \ll |\mathbf{x}|, \ |\mathbf{x}-\mathbf{x}'| = r$

**weak source**
**long wavelengths**
**large r**

$$\bar{h}_{jk}(t,\mathbf{x}) = \frac{2}{r}\frac{d^2 I_{jk}(t-r)}{dt^2}$$

$$I_{jk} = \int \rho x^j x^k d^3x$$

**conservation of energy momentum tensors are used**

$$\frac{\partial T^{tt}}{\partial t} + \frac{\partial T^{kt}}{\partial x^k} = 0, \text{ and } \frac{\partial^2 T^{tt}}{\partial t^2} = \frac{\partial^2 T^{kl}}{\partial x^k \partial x^l}$$

# Order of magnitude estimates of GW amplitude

$$I_{jk} = \int \rho x^j x^k d^3x \qquad Q_{jk} = \int \rho \left( x^j x^k - \frac{1}{3} r^2 \delta_{jk} \right) d^3x$$

**Note: the moment of inertia tensor is not exactly the same as the quadrupole moment tensor, but in TT guage, it does not matter.**

$$\ddot{I}_{jk} \sim \frac{(\text{mass}) \times (\text{size})^2}{(\text{transit time})^2} \sim \text{quadrupole Kinetic E.} = \epsilon M c^2$$

$$h_{jk} \sim 10^{-22} \left( \frac{\epsilon}{0.1} \right) \left( \frac{M}{M_\odot} \right) \left( \frac{100\text{Mpc}}{r} \right)$$

**In most cases, $\epsilon$ is small, but it could become ~0.1**



# Quadrupole radiation from a binary NS merger

$$h \sim \frac{G}{c^4} \frac{E^{\text{ns}}_{\text{kin}}}{r}$$

$$E^{\text{ns}}_{\text{kin}} \sim 1.4 M_\odot (\pi a / T)^2$$

$$T^2 = \left( \frac{4\pi^2}{GM} \right) a^3$$

$$h \sim 10^{-20} \times \left( \frac{6.3 \text{ kpc}}{r} \right) \left( \frac{M}{2.8 M_\odot} \right)^{5/3} \left( \frac{T}{1.0\text{s}} \right)^{-2/3}$$

## 2. GW sources:

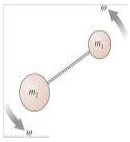
$$h_{\mu\nu} = \frac{2G}{Rc^4}\ddot{I}_{\mu\nu},$$

$1.6 \times 10^{-44}\ \mathrm{sec}^2\mathrm{kg}^{-1}\mathrm{m}^{-1}$

So, extremely weak for most cases!

- Laboratory generation of GWs:

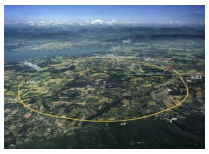
Ex). A rotating dumbbell of two masses: 1ton, 2m & 1kHz



$$h_{lab} = 2.6 \times 10^{-33}\mathrm{m} \times \frac{1}{R}.$$

R ~ λ = 300km ➔ $h_{lab} = 9 \times 10^{-39}$

Ex). Particle accelerators, e.g., LHC:



$$h_{\mu\nu} \sim 4.94 \times 10^{-54} \frac{\frac{m}{m_p}\left(\frac{v}{v_0}\right)^2}{r} \quad \text{where } v_0 = 0.9$$

$$\sim 10^{-30} \frac{(v/v_0)^2}{r} \quad \text{for } m = 1.67g = 10^{24} m_p$$

In the LHC, v~0.999999991 and 10^11 protons per bunch ➔ h ~ 10^-43 !!

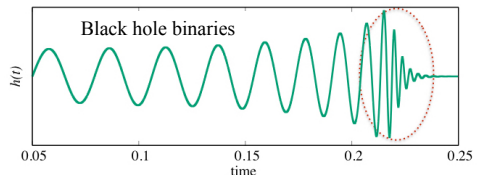


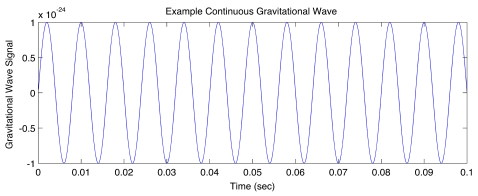
# GW sources

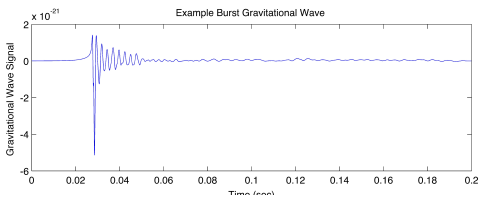
- **Compact Binary Coalescence** (neutron stars and black holes)
  - Strong signal, but rare
  - Computable waveforms
- **Continuous** (~ single neutron stars)
  - Weak, but could be abundant
  - Deviation from axis-symmetry is not known
- **Burst** (supernovae or gamma-ray bursts)
  - GW amplitudes are now well known, not frequent

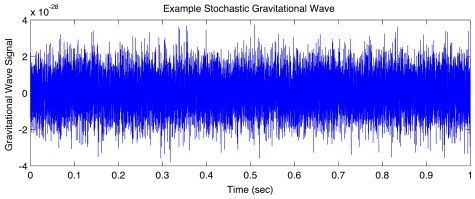
$$I_{jk} = \int \rho x^j x^k d^3x$$

- **Stochastic**
  - Superposition of many random sources
  - Could be useful to understand distant populations of GW sources or early universe

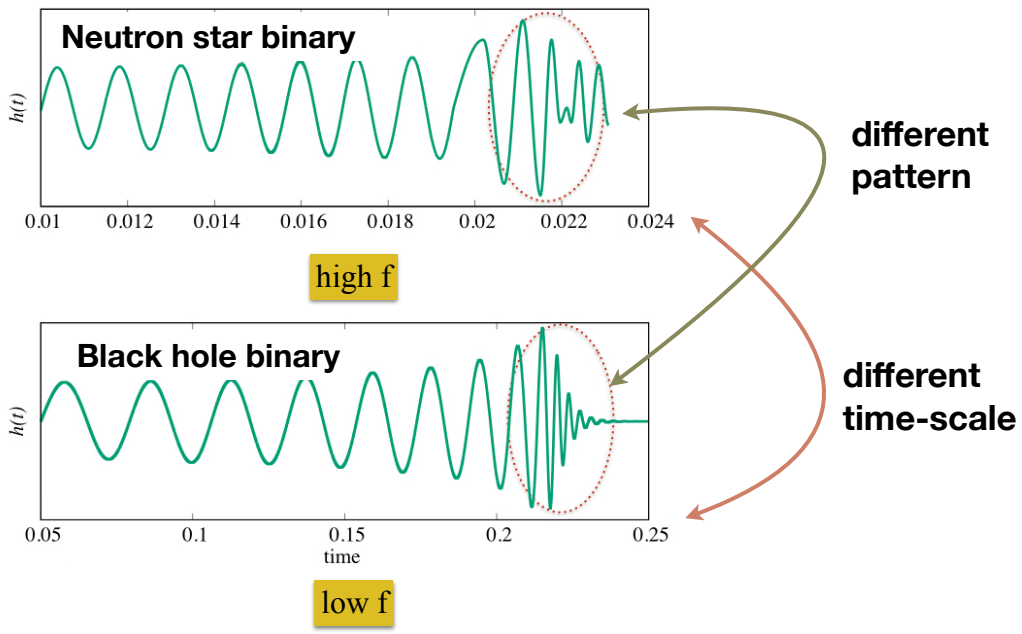


# Waveforms can tell many things

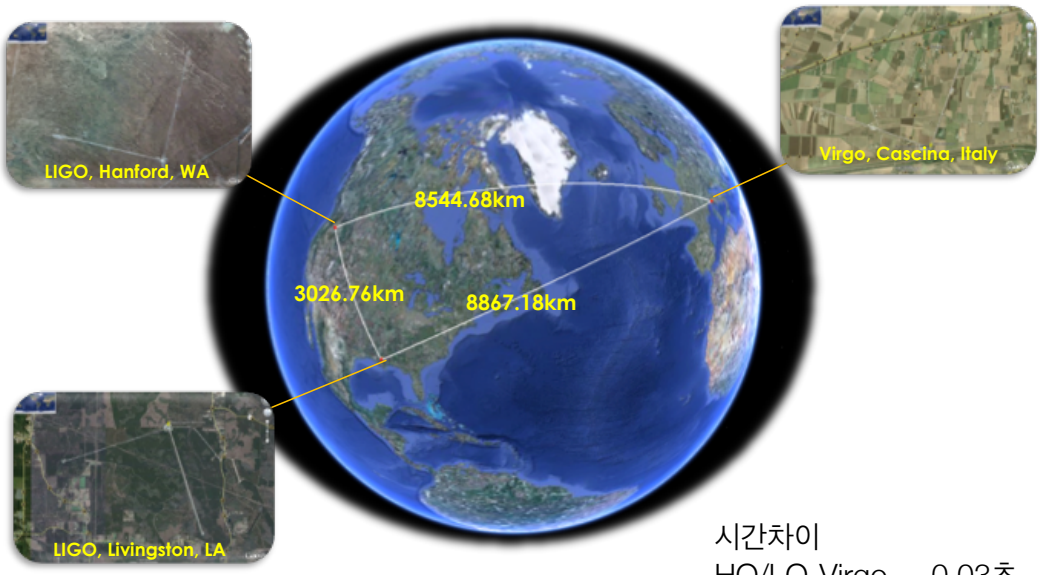





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- **LIGO**
- Neutron star coalescence and GW
- Prospects

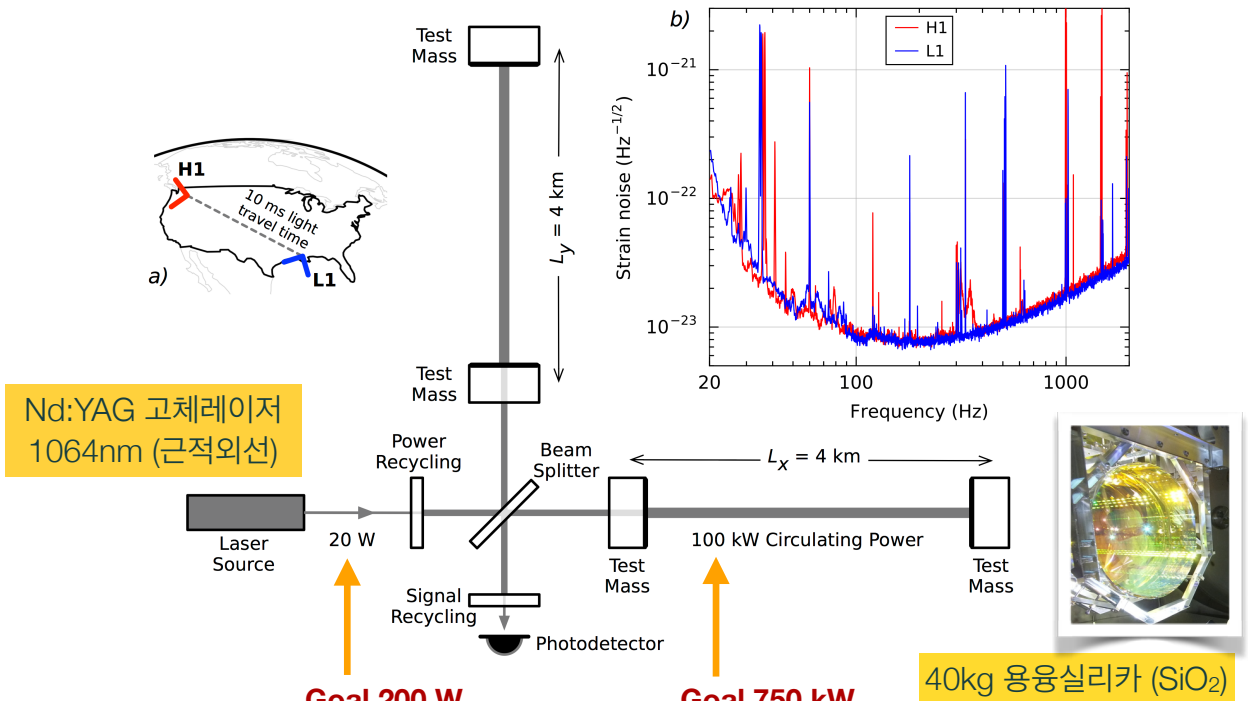
# 중력파 검출기 네트워크




시간차이
HO/LO-Virgo ~ 0.03초
HO-LO ~ 0.01초



Laser Interferometer Gravitational-wave Observatory

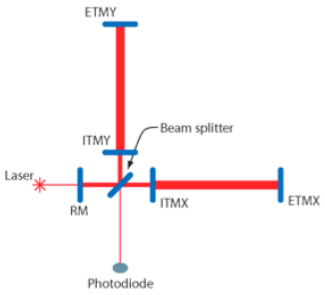


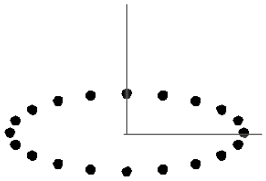

Nd:YAG 고체레이저
1064nm (근적외선)



40kg 용융실리카 ($SiO_2$)
(흡수율 1ppm 이하)

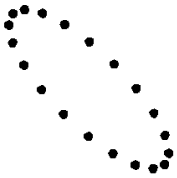
# **GW** propagating in z-direction



$h_+$

**plus polarisation**



$h_X$

**cross polarisation**



## What is interacting with GW, laser light or test mass?

**Kip Thorne**

- **TT gauge**
  - mirrors & beam splitters remain at rest
  - GW (h) interact with interferometer's light
- **local Lorentz (LL) frame**
  - in the proper reference frame of beam splitter
  - beam splitter is freely falling with GW
  - mirrors are moving
  - tiny correction due to GW & light interaction
- **physical observables should be independent of coord choice**

# 라이고 검출기 성능

- 4km 거리에서 양성자 직경의 1000분의 1 변화 측정 가능
  (1광년 거리에서 머리카락 굵기의 1/10 변화에 해당)
- 강한 레이저 파워 (20 W ➡ 700 W ➡ 100 kW)
  (최종 목표 : 200 W ➡ 750 kW)
- 거울 사이의 레이저 약 280번 반복하여 4km 거리 한계 극복
  (1120 km 효과)
- 검출 가능 거리 :
  중성자별 충돌은 약 10억 광년 / 블랙홀 충돌은 약 30억 광년



# Sensitivity of Interferometers

**If the length resolution is $\lambda_{laser}$, detectable strain is**

$$h \equiv \frac{\Delta l}{l} = \frac{\lambda_{laser}}{l} = \frac{10^{-6}\mathrm{m}}{10^{3}\mathrm{m}} = 10^{-9}$$

**Fabry-Perot cavity (~ 250 bounces) :** $\lambda_{\mathrm{GW}} \sim 1000\ \mathrm{km}$ for $300\ \mathrm{Hz}$

$$h \sim \frac{\Delta l}{l_{eff}} \sim \frac{\lambda_{laser}}{\lambda_{GW}} \sim \frac{10^{-6}\mathrm{m}}{10^{6}\mathrm{m}} = 10^{-12}$$

**Due to quantum nature of the photons, the length resolution could be reduced**

$$h \sim \frac{1}{\sqrt{N_{photons}}} \frac{\lambda_{laser}}{\lambda_{GW}}$$

## Measuring the phase shift by averaging over some period τ

Poisson distribution

$$p(N) = \frac{\bar{N}^N \exp\left(-\bar{N}\right)}{N!}, \quad \Delta N = \sqrt{\bar{N}}$$

N: measured number of photons over period τ

$$E = \hbar \frac{2\pi c}{\lambda_l} N \quad \rightarrow \quad \Delta E = \hbar \frac{2\pi c}{\lambda_l} \sqrt{\bar{N}}$$

Uncertainty in the phase measurement : $\Delta\Phi = 2\pi c \frac{\Delta t}{\lambda_l}$

$$\Delta t \Delta E \geq \frac{\hbar}{2} \quad \rightarrow \quad \Delta t \Delta E = \frac{\Delta\Phi \lambda_l}{2\pi c} \hbar \frac{2\pi c}{\lambda_l} \sqrt{\bar{N}} \geq \frac{\hbar}{2}$$

$$\rightarrow \quad \Delta\Phi \geq \frac{1}{\sqrt{\bar{N}}} \quad \rightarrow \quad N \geq \frac{1}{2(\Delta\Phi)^2}$$



## Shot noise

한국 중력파 연구 협력단
Open Collaboration for the Gravitational Wave Detection and its Application

**Collect photons for a time of the order of the period of GW wave**

$$\tau \sim 1/f_{GW}$$

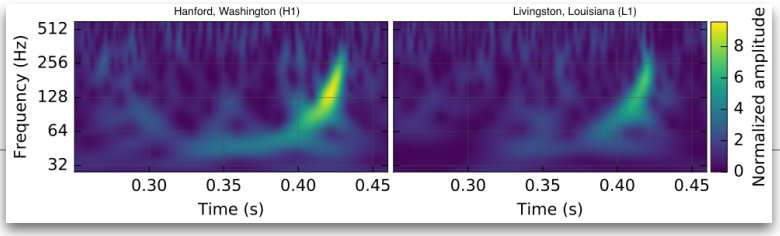
$$N_{photons} = \frac{P_{laser}}{hc/\lambda_{laser}} \tau \sim \frac{P_{laser}}{hc/\lambda_{laser}} \frac{1}{f_{GW}}$$

**For 1W laser with $\lambda_{laser}$=1 μm, $f_{GW}$=300Hz, $N_{photons}$=$10^{16}$**

$$h \sim \frac{\Delta l}{l_{eff}} \sim \frac{N_{photons}^{-1/2} \lambda_{laser}}{\lambda_{GW}} \sim \frac{10^{-8} \times 10^{-6}\mathrm{m}}{10^{6}\mathrm{m}} = 10^{-20}$$

**By adopting high power laser (20W for O1) and Power Recycling, we can reach 'astrophysical sensitivity' of ~$10^{-22}$.**

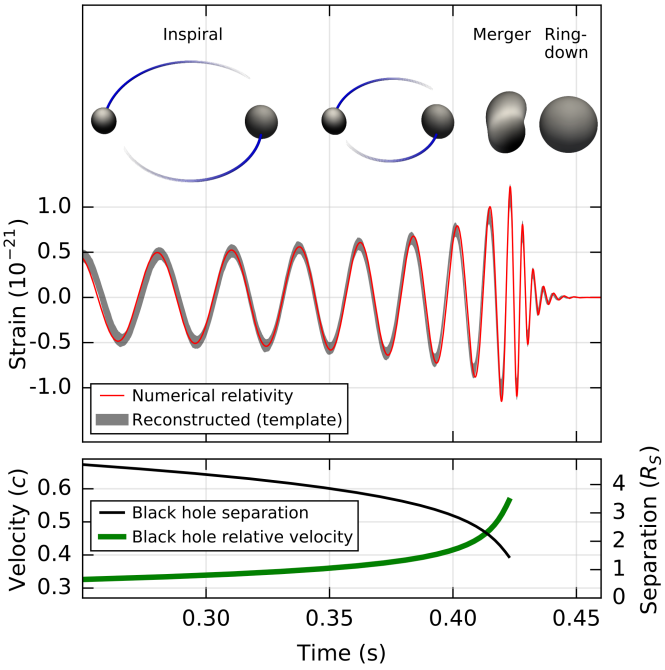
# Gw150914







LSC LIGO Scientific Collaboration VIRGO






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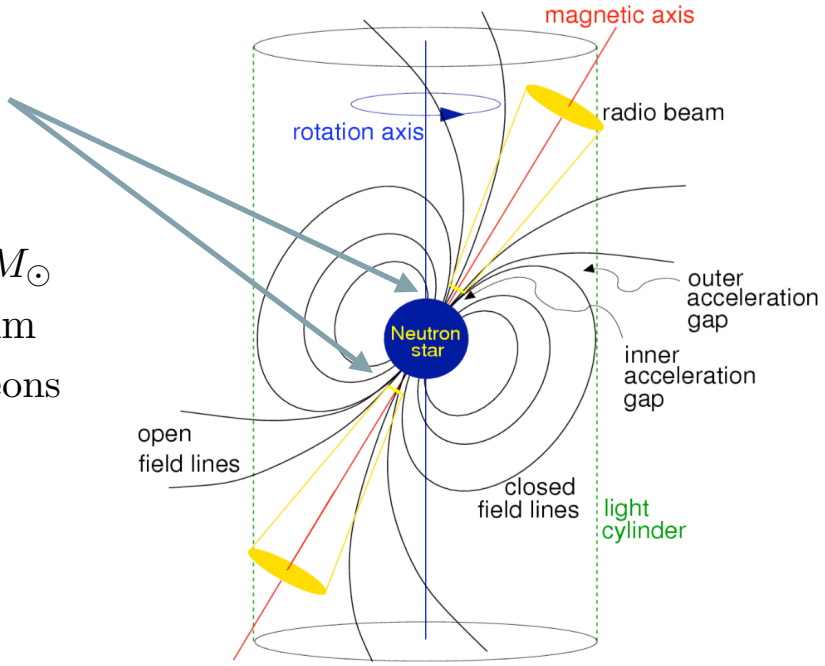
- Motivation
- Spacetime and GW
- Principle of general relativity & GW
- LIGO
- **Neutron star coalescence and GW**
- Prospects

# Why Neutron Stars ?

## Ultimate testing place for physics of dense matter

$e^+e^-$ pair creation

$$M = 1.5 \sim 2.0 M_\odot$$
$$R = 10 \sim 15 \text{ km}$$
$$A \sim 10^{57} \text{ nucleons}$$






*Theomodynamics with General Relativity*

$$P = P(\rho, T, \text{composition})$$
$$\kappa = \kappa(\rho, T, \text{composition})$$
$$\epsilon = \epsilon(\rho, T, \text{composition})$$

**Hydrostatic equilibrium**

$$\frac{dP}{dr} = -\frac{GM\rho}{r^2} \qquad \frac{dP}{dr} = -\frac{GM\rho}{r^2}\left(1+\frac{P}{\rho c^2}\right)\left(1+\frac{4\pi P r^3}{Mc^2}\right)\left(1-\frac{2GM}{rc^2}\right)^{-1}$$

**Mass continuity**

$$\frac{dM}{dr} = 4\pi r^2 \rho \qquad \frac{dM}{dr} = 4\pi r^2 \left(\frac{\epsilon}{c^2}\right)$$

includes all energy sources

**physics of dense nuclear matter (strong interaction)**

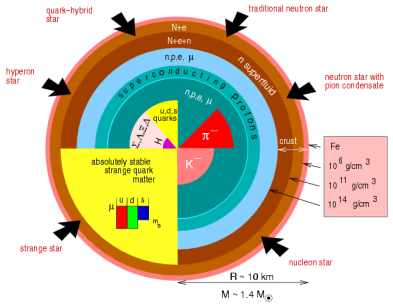
**Radiative energy transport**

$$\frac{dT(r)}{dr} = -\frac{3L(r)\kappa(r)\rho(r)}{4\pi r^2 4acT(r)^3}$$

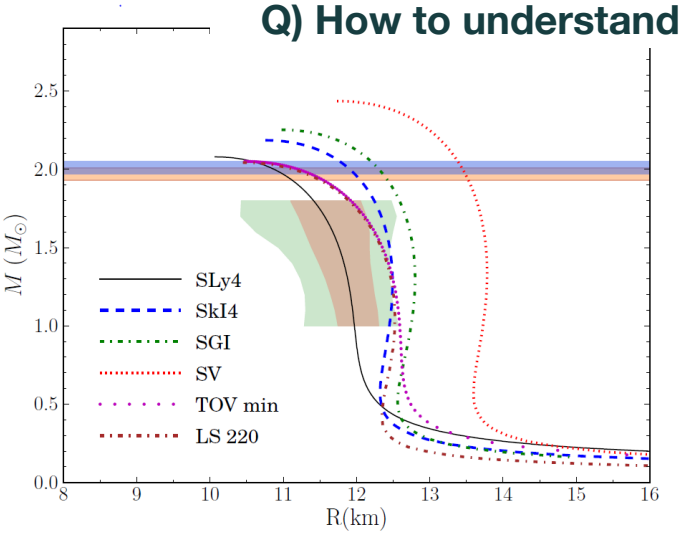
**Energy conservation**

$$\frac{dL(r)}{dr} = 4\pi r^2 \rho(r)\epsilon(r)$$

# Maximum Mass of Neutron Stars




**Q) How to understand ?**




**Neutron Star-White Dwarf Binaries**

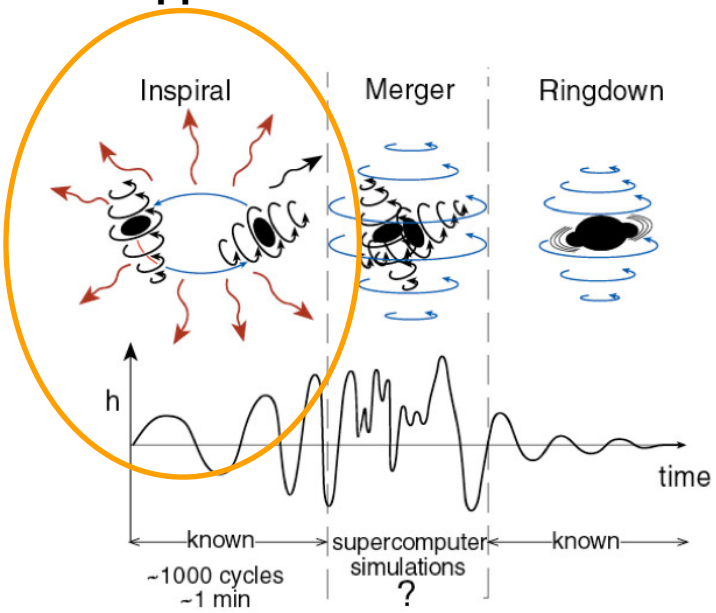
1.97 solar mass NS : Nature 467 (2010) 1081

2.01 solar mass NS : Science 340 (2013) 6131



# Response of NS to GW during Inspiral

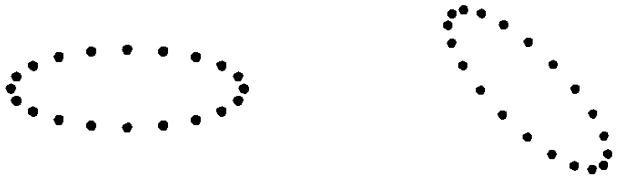
**perturbative approaches**



# Tidal deformability & Love number

## Selected references

- **A.E.H. Love** (1909) - The Yielding of the Earth to Disturbing Forces
- **K.S. Thorne** & A. Campolattaro (1967) - non-radial pulsation of NS
- J.B. Hartle & **K.S. Thorne** (1969) - stability of rotating NS
- … …
- **K.S. Thorne** (1998) - Tidal stabilization of rigid rotating, fully relativistic neutron star
- … …





# Tidal deformability & Love number

$$-\frac{(1+g_{tt})}{2} = -\frac{m}{r} - \frac{3Q_{ij}}{2r^3}\left(n^i n^j - \frac{1}{3}\delta^{ij}\right) + \mathcal{O}\left(\frac{1}{r^3}\right) + \frac{\mathcal{E}_{ij}}{2} r^2 n^i n^j + \mathcal{O}(r^3)$$

$\mathcal{E}_{ij}$ : external quadrupole tidal field

$Q_{ij}$ : quadrupole moment of NS

$\lambda$ : Tidal deformability

$$Q_{ij} = -\lambda \mathcal{E}_{ij}$$

$$Q_{ij} = \int d^3x \delta\rho(x)\left(x_i x_j - \frac{1}{3}r^2\delta_{ij}\right)$$

$$n^i = \frac{x^i}{r}$$

**dimensionless parameter**

$k_2$ : $l = 2$ Tidal Love number

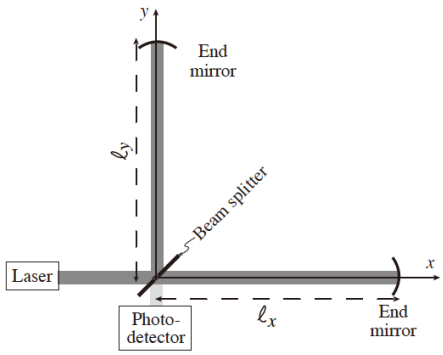
$$k_2 = \frac{3}{2} G\lambda R^{-5}$$

Hinderer et al. PRD 81 (2010)

# Laser Interferometer

- Consider a simple Michelson interferometer with $l_x \approx l_y \approx l$.
- The phase difference of returning lights reflected by x and y ends

$$\Delta\phi = \phi_x - \phi_y \approx 2\omega_0 \left[l_x - l_y + lh(t)\right]$$




- Toward the photo-detector:

$$E_{PD} \propto e^{i\phi_x} - e^{i\phi_y} = e^{i\phi_y}(e^{i\Delta\phi} - 1)$$

- For small $\Delta\phi$,

Time varying part

$$I_{PD} \propto |e^{i\Delta\phi} - 1|^2 \approx |\Delta\phi|^2 \approx 4\omega_0^2(l_x - l_y)^2 + 8\omega_0^2(l_x - l_y)lh(t)$$



PRL 112, 101101 (2014) PHYSICAL REVIEW LETTERS week ending 14 MARCH 2014

**Systematic Parameter Errors in Inspiraling Neutron Star Binaries**

Marc Favata*

$$\Psi_{\mathrm{T}}(f) = \varphi_c + 2\pi f t_c + \frac{3}{128\eta v^5}\left(\Delta\Psi^{\mathrm{pp}}_{3.5\mathrm{PN}} + \Delta\Psi^{\mathrm{spin}}_{3\mathrm{PN}} + \Delta\Psi^{\mathrm{ecc.}}_{2\mathrm{PN}} + \Delta\Psi^{\mathrm{tidal}}_{6\mathrm{PN}} + \Delta\Psi^{\mathrm{tm}}_{6\mathrm{PN}}\right)$$

$$v = (\pi f M)^{1/3}$$

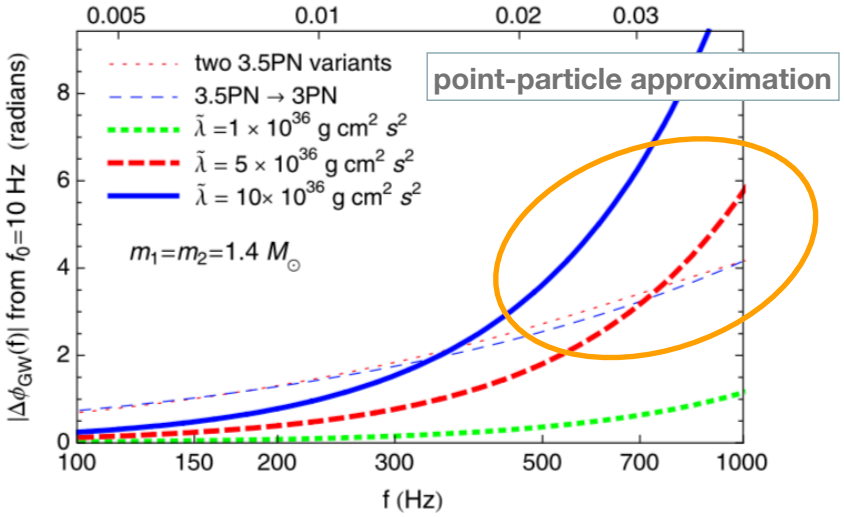
$$v/c = (GM\pi f/c^3)^{1/3}$$

$$\Delta\Psi^{\mathrm{tidal}}_{6\mathrm{PN}} = -\frac{39}{2}\tilde{\Lambda}v^{10} + v^{12}\left(\frac{6595}{364}\delta\tilde{\Lambda} - \frac{3115}{64}\tilde{\Lambda}\right)$$

$$\tilde{\Lambda} \equiv 32\frac{\tilde{\lambda}}{M^5} = \frac{8}{13}[(1 + 7\eta - 31\eta^2)(\hat{\lambda}_1 + \hat{\lambda}_2) - \sqrt{1 - 4\eta}(1 + 9\eta - 11\eta^2)(\hat{\lambda}_1 - \hat{\lambda}_2)].$$

# accumulated GW phase

$$|\Delta\phi_{\rm GW}(f)| = |\Phi_{3.5,\rm pp}(f_{\rm GW}) - \Phi_{3.5,\lambda}(f_{\rm GW})|$$






PRL 112, 101101 (2014) PHYSICAL REVIEW LETTERS week ending 14 MARCH 2014

**Systematic Parameter Errors in Inspiraling Neutron Star Binaries**

Marc Favata*

# phase shift vs deformability

$$\left.\frac{d\Phi}{dx}\right|_{\rm T} = -\frac{195}{8}\frac{x^{3/2}}{\eta}\frac{\tilde{\lambda}}{M^5} \propto \frac{\tilde{\lambda}}{M^5}$$

$$x = (\omega M)^{2/3} \Rightarrow \left(\omega\frac{GM}{c^3}\right)^{2/3}$$

$$\eta = m_1 m_2/M^2$$

dimensionless

$$\Lambda = \frac{\lambda}{m^5} \Rightarrow G\lambda\left(\frac{c^2}{Gm}\right)^5 \approx 950.5\left(\frac{m_\odot}{m}\right)^5\left(\frac{\lambda}{10^{36}\ {\rm g\ cm^2\ s^2}}\right)$$

$$\Lambda = G\left(\frac{c^2}{Gm}\right)^5 \times \frac{2}{3}\frac{R^5}{G}k_2 = \frac{2}{3}\left(\frac{Rc^2}{Gm}\right)^5 k_2 \approx 9495\left(\frac{R_{10\rm km}}{m_{M_\odot}}\right)^5 k_2$$

Press Release Oct 16, 2017

# GW from Binary NS Mergers

*GW 170817 (**d=40 Mpc**)*
*GRB 170817A by Fermi-GBM*
*Kilonova/X-ray/Optical Afterglows*

PRL **119,** 161101 (2017) Selected for a Viewpoint in *Physics* PHYSICAL REVIEW LETTERS week ending 20 OCTOBER 2017

## GW170817: Observation of Gravitational Waves from a Binary Neutron Star Inspiral

B. P. Abbott *et al.*[*]
(LIGO Scientific Collaboration and Virgo Collaboration)
(Received 26 September 2017; revised manuscript received 2 October 2017; published 16 October 2017)

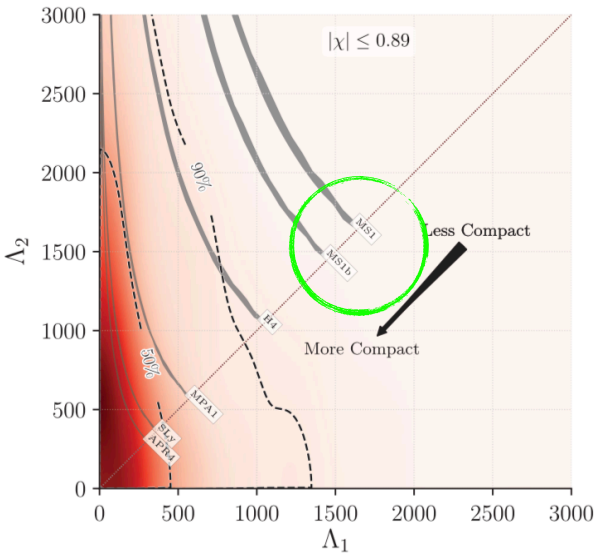


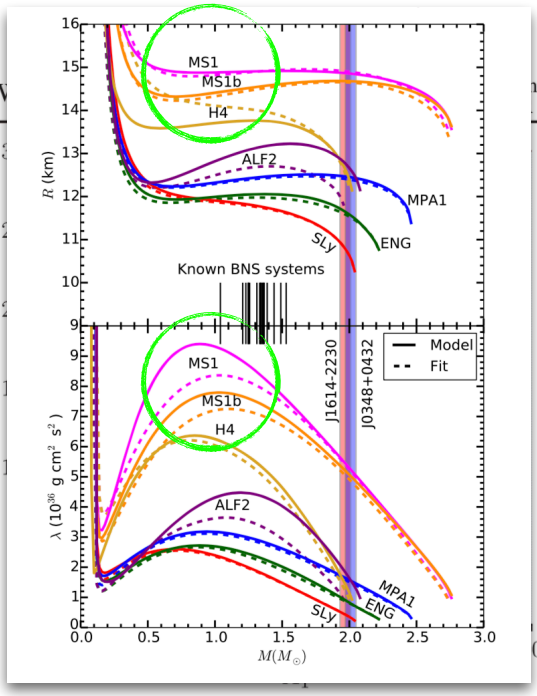
TABLE I. Source properties for GW170817: we give ranges encompassing the 90% credible intervals for different assumptions of the waveform model to bound systematic uncertainty. The mass values are quoted in the frame of the source, accounting for uncertainty in the source redshift.

| | Low-spin priors ($\lvert\chi\rvert \leq 0.05$) | High-spin priors ($\lvert\chi\rvert \leq 0.89$) |
|---|---|---|
| Primary mass $m_1$ | 1.36–1.60 $M_\odot$ | 1.36–2.26 $M_\odot$ |
| Secondary mass $m_2$ | 1.17–1.36 $M_\odot$ | 0.86–1.36 $M_\odot$ |
| Chirp mass $\mathcal{M}$ | $1.188^{+0.004}_{-0.002} M_\odot$ | $1.188^{+0.004}_{-0.002} M_\odot$ |
| Mass ratio $m_2/m_1$ | 0.7–1.0 | 0.4–1.0 |
| Total mass $m_{\rm tot}$ | $2.74^{+0.04}_{-0.01} M_\odot$ | $2.82^{+0.47}_{-0.09} M_\odot$ |
| Radiated energy $E_{\rm rad}$ | $> 0.025 M_\odot c^2$ | $> 0.025 M_\odot c^2$ |
| Luminosity distance $D_{\rm L}$ | $40^{+8}_{-14}$ Mpc | $40^{+8}_{-14}$ Mpc |
| Viewing angle $\Theta$ | $\leq 55°$ | $\leq 56°$ |
| Using NGC 4993 location | $\leq 28°$ | $\leq 28°$ |
| Combined dimensionless tidal deformability $\tilde{\Lambda}$ | $\leq 800$ | $\leq 700$ |
| Dimensionless tidal deformability $\Lambda(1.4 M_\odot)$ | $\leq 800$ | $\leq 1400$ |

$$\tilde{\Lambda} = \frac{16}{13}\frac{(m_1 + 12m_2)m_1^4\Lambda_1 + (m_2 + 12m_1)m_2^4\Lambda_2}{(m_1 + m_2)^5}$$





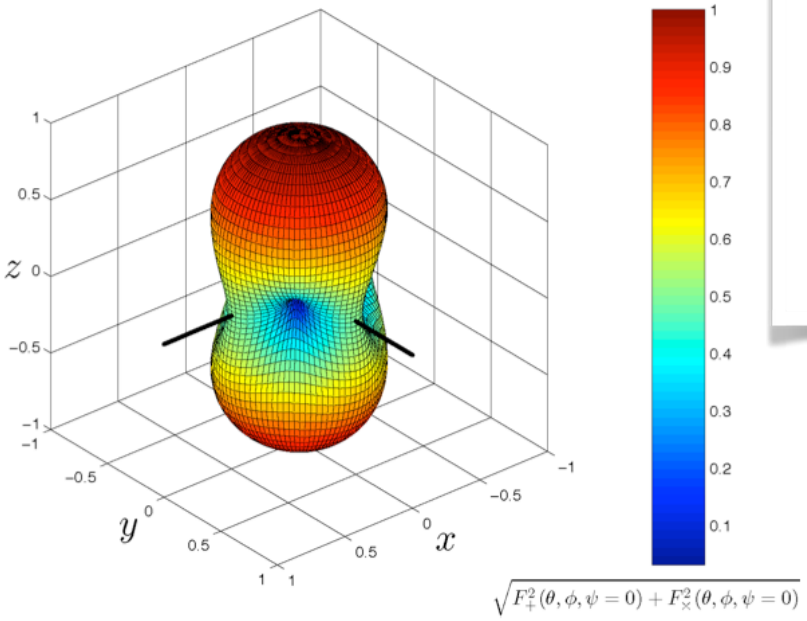



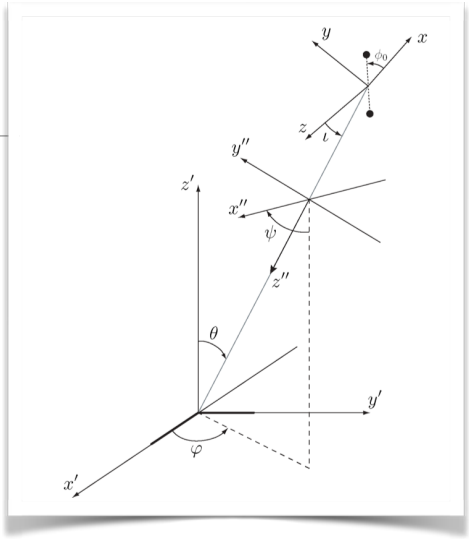



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# Magnitude of strain




with $\iota = \psi = 0$



## in detector's frame $(r, \theta, \varphi)$

$$h(t) = F_+ h_+(t) + F_\times h_\times(t)$$

$$F_+ = -\frac{1}{2}(1+\cos^2\theta)\cos 2\varphi \cos 2\psi - \cos\theta \sin 2\varphi \sin 2\psi$$

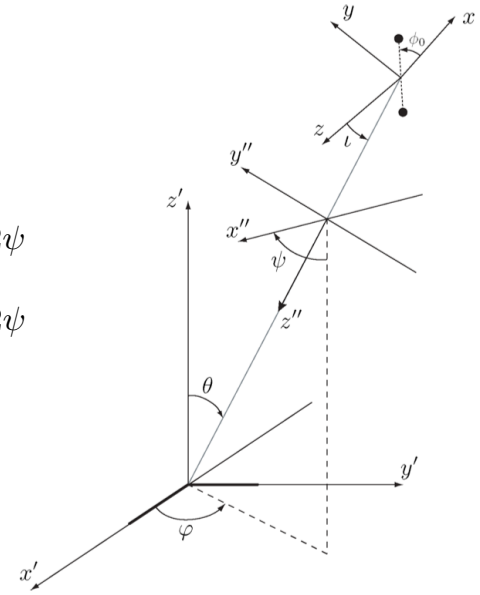
$$F_\times = +\frac{1}{2}(1+\cos^2\theta)\cos 2\varphi \sin 2\psi - \cos\theta \sin 2\varphi \cos 2\psi$$

$$h_+(t) = -\frac{GM_\odot}{c^2 r}(1+\cos^2\iota)h_c(t)$$

$$h_\times(t) = -\frac{2GM_\odot}{c^2 r}(\cos\iota)h_s(t)$$

$$h_c(t) = \frac{2}{c^2}\left(\frac{\mu}{M_\odot}\right)[\pi G M f(t)]^{2/3}\cos[2\phi(t) - 2\phi_0]$$

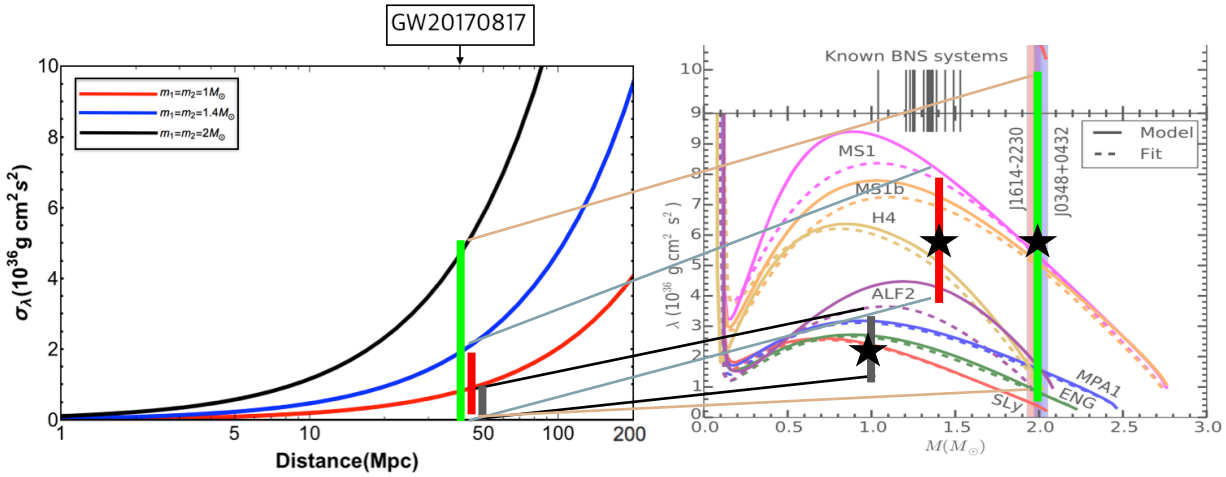
$$h_s(t) = \frac{2}{c^2}\left(\frac{\mu}{M_\odot}\right)[\pi G M f(t)]^{2/3}\sin[2\phi(t) - 2\phi_0]$$




**optimal condition** $\iota = 0 \;\&\; \theta = 0$

## Uncertainty in the estimation of deformability **under optimal condition**

**With Y-B Choi, H-S Cho, YM Kim**




GWs from low-mass NS mergers are better to distinguish NS EoS



## Prospects

- **Distribution of black holes and neutron stars**
- **Tidal deformability and NS structure from GW**
- **From 2019, expecting more GWs from aLIGO**